1 Introduction

About this lecture

- Argument forms
- Logical operators
- Formalization: well formed formula (wff)
- Truth table
- Course homepages: http://mathsci.kaist.ac.kr/~schoi/logic.html and the moodle page http://moodle.kaist.ac.kr
- Grading and so on in the moodle. Ask questions in moodle.

Some helpful references

- Richard Jeffrey, Formal logic: its scope and limits, Mc Graw Hill
- A mathematical introduction to logic, H. Enderton, Academic Press.
- Whitehead, Russel, Principia Mathematica (our library). (This could be a project idea.)
- http://plato.stanford.edu/contents.html has much resource.
- http://ocw.mit.edu/OcwWeb/Linguistics-and-Philosophy/24-241Fall-2005/CourseHome/

2 Argument forms

Argument forms

- P, Q, R represent some sentences (not nec. atomic)
- Either today is Monday or Tuesday.
- P or Q.
- If you have bad grades in KAIST, then you pay tuition.
- If P, then Q.
- If P and Q, then R. It is not the case R. It is not the case P and Q.

3 Logical operators

Logical operators

- It is not the case that: $\neg$ or ˜
- And: $\wedge$ or &
- Or: $\vee$
- If ..., then... : $\rightarrow$.
- If and only if: $\leftrightarrow$.
- See `http://plato.stanford.edu/entries/pm-notation/`.
- See `http://en.wikipedia.org/wiki/Logical_connective/`.
- $\vdash$ is used to mark the conclusion.

Precedence

- The order of precedence determines which connective is the "main connective" when interpreting a non-atomic formula.
- As a way of reducing the number of necessary parentheses, one may introduce precedence rules:
- Operator Precedence
  - $\neg$
  - $\wedge$
  - $\vee$
  - $\rightarrow$
  - $\leftrightarrow$
- So for example, $P \vee Q \wedge \neg R \rightarrow S$ is short for
- $(P \vee (Q \wedge (\neg R))) \rightarrow S$.

Formalizations

- We can formalize any sentence by dividing it into atomic parts.
- It is not both raining and snowing.
- $\neg(R \wedge S)$
- It is neither raining nor snowing.
- $\neg R \vee \neg S$

4 Well formed formula or wff

Well formed formula or wff

- $((\wedge P) \vee Q \neg R)$ This is a nonsense
- We define this inductively.
  - Any sentence letter is wff. (atomic one)
  - If ϕ is wff, then so is $\neg\phi$.
  - If ϕ and ψ are wff, so is $(\phi \wedge \psi)$, $(\phi \vee \psi)$, $(\phi \rightarrow \psi)$, and $(\phi \leftrightarrow \psi)$.
- subwff is a wff within a wff.
- As long as atomic sentence letters are well defined, there is no ambiguity in the meaning of wff.

Some exercises

- Either there is no Starbuck's in Daejeon or I do not buy coffee bins.
- $\neg S \vee \neg B$.
- If I buy coffee bins, then there is no Starbuck's in Daejeon.
- $B \rightarrow \neg S$.
- If there were no God, then no movement is possible. But there are movements. Hence, God exists.
- $\neg G \rightarrow \neg M, M, \vdash G$.

Some exercises

- Either it is raining, or it's both snowing and raining.
- $R \vee (R \wedge S)$.
- Either it is both raining and snowing or it is snowing but not raining.
- $(R \wedge S) \vee (S \wedge \neg R)$.

5 Truth table

Semantics of the logical operators

- semantics: the study of meaning.
- Each atomic formula has a truth or false value in a real world (or world A).
- Each wff has truth or false value in a real world (or world A).
- This depends on the truth value of atomic formulas.

Truth tables

- Truth table generator:
  - http://en.wikipedia.org/wiki/Truth_table,
  - http://logik.phl.univie.ac.at/~chris/gateway/formular-uk-zentral.html, complete (i.e. has all the steps)
  - http://svn.oriontransfer.org/TruthTable/index.rhtml, has xor, complete.
  - One has to learn some notations... Sometimes use 0 and 1 instead of F and T.

Truth tables

- Elementary ones:
  - $\neg a$
  - $a \wedge b$
  - $a \vee b$
  - $a \rightarrow b$.
  - $a \leftrightarrow b$ or $(a \rightarrow b) \wedge (b \rightarrow a)$
  - Every wff can be evaluated from this.
  - In computer science xor.

Examples

- To construct a truth table for a complex wff, we find the truth values for its smallest subwffs and then use the truth tables for the logical operators for larger subwff and so on....
- $\neg S \wedge \neg B$.
- $(\neg G \rightarrow \neg M) \rightarrow (M \rightarrow G)$.
- Also compare $P \rightarrow Q$ and $\neg P \vee Q$. Check$(P \rightarrow Q) \leftrightarrow (\neg P \vee Q))$.
- This is used to compare.
- You can also use $\neg((P \rightarrow Q)$ xor $(\neg P \vee Q))$.

Tautology and a contradiction

- Given some formula, any assignment of T and F yields T in the truth table. Such a formula is said to be a *tautology*.
- $P \vee \neg P$.
- Given some formula, any assignment of T and F yields F in the truth table. Such a formula is said to be a *contradiction*. (*truth-functionally inconsistent*)
- $P \wedge \neg P$.
- The formula which are not one of the above is said to be *truth-functionally contingent*.

Examples

- $(\neg G \to \neg M) \to (M \to G)$.
- $(P \to Q) \leftrightarrow (\neg P \vee Q))$.
- $((P \to Q) \to R) \to (P \to R)$.

Truth table for argument forms

- Here, we will have a number of premises $P_1, P_2, .$ and a conclusion Q. We need to find the validity of $P_1, P_2, ... \vdash Q$
- P_i s are complex.
- To check validity... We check when if every P_i is true, then so is Q.
- Or you can form $(P_1 \wedge P_2 \wedge \cdots \wedge P_n) \to Q$.

Examples

- $P \to Q, P \to \neg Q \vdash \neg P$.
- $((P \to Q) \wedge (P \to \neg Q)) \to \neg P$.
- $R \vdash P \leftrightarrow (P \vee (P \wedge Q))$.
- $R \to (P \leftrightarrow (P \vee (P \wedge Q)))$.