ERRATA IN "GEOMETRIC STRUCTURES ON LOW-DIMENSIONAL MANIFOLDS"

SUHYOUNG CHOI

Theorem 7 of [1] in page 338 is not correct. The correct version is as follows:

Theorem 0.1. *Let T be a 3-orbifold with base space homeomorphic to a tetrahedron with some vertices removed. Suppose that the faces are silvered and edges have local group conjugate to dihedral group action generated by reflections on two 2-planes in $\mathbb{R}^3$. The edge order is the half of the dihedral group order. Also, vertices of any order 2-edges are not removed. Then the deformation space of projective structures is homeomorphic to a cell of dimension $3 - e_2$ where e_2 is the number of edges of order 2.*

Proof. The dimension follows from Theorem 4 of [2]. Let T be a tetrahedron with vertices v_i and opposite face F_i and edges e_{ij} in $F_i \cap F_j$ with orders n_{ij}. Put a tetrahedron in $\mathbb{R}P^3$ so that the vertices are at $[1,0,0,0], [0,1,0,0], [0,0,1,0]$, and $[0,0,0,1]$. Each F_i corresponds to a projective reflection fixing points on the hyperspace containing F_i and a reflection point v_i. The direct computations as in Propsition 4 of [2] proves the result: Any order 2 edge meeting faces F_i and F_j implies that the reflection point v_i of F_i has the j-coordinate $(v_i)_j = 0$ and $(v_j)_i = 0$. The equation we need to solve is $(v_i)_j(v_j)_i = 2\cos\pi/n_{ij}$ if for each edge with order $n_{ij} > 2$ and $(v_i)_j = (v_j)_i = 0$ if for each edge with order$n_{ij} = 2$. Thus, this is just a system of purely multiplicative equations to solve. (This was already carried out by J.R. Kim in 1998 following Goldman, Kac-Vinberg.) (In the compact case, the result follows from Theorem 3.16 [3].) □

On page 339, the pyramid P has the deformation space homeomorphic to a cell of dimension 2. This can be proved as follows: Put any pyramid as in page 82 of [2]. Then reflection points $v_1, .., v_4$ of triangular sides have to lie on two lines through the top vertex. If we are assigned a reflection point v_5 for a bottom vertex, then $v_1, ..., v_4$ are determined by equations as above. Since the group of projective automorphism fixing P is fixes the top vertex and each point of the bottom

Date: November 3, 2011.

side, it is isomorphic to $\mathbb{R}$. Thus, we choose v_4 to lie on a hyperspace H disjoint from P. One can show H intersected with the open half-spaces formed from faces of P meeting the top faces is the space of possible choice of v_4. Thus, we obtain a 2-cell as a space of deformations.

For an octahedron O with all edge orders 2, the deformation space is homeomorphic to a 3-cell. This follows since the reflection point v_i of F_i is determined by the hyperspaces of the adjacent faces. Thus, the octahedron determines all the reflection points. The projective congruence space of the space of octahedrons in $\mathbb{R}P^3$ can be considered a subspace of the projective congruence space of six points where we allow collinearity of four points. We take the top and the bottom vertices and three of the vertices in the middle. This give five vertices in general position. The remaining vertex can be chosen inside a convex domain determined by the five vertices. Thus, the deformation space is parametrized by this 3-cell.

References

[1] S. Choi, *Geometric structures on low-dimensional manifolds*, J. Korean Math. Soc. 40 (2003) no. 2, pp. 319–340.

[2] S. Choi, *The deformation space of projective structures on 3-dimensional Coxeter orbifolds*, Geom. Dedicata 119 (2006), 69–90.

[3] L. Marquis, *Espaces des modules de certains polyèdres projetifs mirroirs* , Geom. Dedicata 147 (2010), 47–86.

Department of Mathematics, KAIST, Daejeon 305-701, South Korea
E-mail address: schoi@math.kaist.ac.kr