Topology: Final Exam (Spring 2007)

Justify your answers fully.

1. Answer the following without proofs:

   (a) (10pts.) Find an example of a Hausdorff but not regular space.

   (b) (10pts.) Find an example of a regular but not normal space.

2. Prove:

   (a) (20pts) A regular Lindelöf space is normal.

   (b) (20pts) A locally compact Hausdorff space is completely regular.

   (c) (20pts) The product space $\mathbf{R}^J$ is normal if J is a countable set.

   (d) (20pts) A one-point compactification of $\mathbf{R}^2$ with standard topology is homeomorphic to the unit sphere S^2 in $\mathbf{R}^3$.

   (e) (20pts) An open subspace of a Baire space is Baire.

3. Describe the one-point compactification of each of the following topological spaces as a subspace of $\mathbf{R}^n$ for some $n = 1, 2, 3$.

   (a) (15pts) $\mathbf{Z}$ with discrete topology.

   (b) (15pts) $\mathbf{R}^2 - \{0\}$ with the standard subspace topology from $\mathbf{R}^2$.