2-orbifolds, triangulations, and topological constructions and covering spaces of orbifolds

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Outline

- Section 3: Topology of 2-orbifolds
  - Topology of 2-orbifolds
  - Smooth 2-orbifolds and triangulations
- Covering spaces
  - Fiber-product approach
  - Path-approach by Haefliger

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Outline

2-orbifolds

- We now wish to concentrate on 2-orbifolds.
- Singularities
  - We simply have to classify finite groups in $O(2)$: $\mathbb{Z}_2$ acting as a reflection group or a rotation group of angle $\pi/2$, a cyclic groups C_n of order ≥ 3 and dihedral groups D_n of order ≥ 4.
  - According to this the singularities are of form:
    - A silvered point
    - A cone-point of order ≥ 2.
    - A corner-reflector of order ≥ 2.

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2-orbifolds

- On the boundary of a surface with a corner, one can take mutually disjoint open arcs ending at corners. If two arcs meet at a corner-point, then the corner-point is a *distinguished one*. If not, the corner-point is *ordinary*. The choice of arcs will be called the *boundary pattern*.
- As noted above, given a surface with corner and a collection of discrete points in its interior and the boundary pattern, it is possible to put an orbifold structure on it so that the interior points become cone-points and the distinguished corner-points the corner-reflectors and boundary points in the arcs the silvered points of any given orders.

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Outline

The triangulations of 2-orbifolds and classification

- One can put a Riemannian metric on a 2-orbifold so that the boundary is a union of geodesic arcs and each corner-reflector have angles π/n for its order n and the cone-points have angles $2\pi/n$.
- Proof: First construct such a metric on the boundary by putting such metrics on the boundary by using a broken geodesic in the euclidean plane and around the cone points and then using partition of unity.
- By removing open balls around cone-points and corner-reflectors, we obtain a smooth surface with corners.
- Find a smooth triangulation of so that the interior of each side is either completely inside the boundary with the corners removed.
- Extend the triangulations by cone-construction to the interiors of the removed balls.

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The triangulations of 2-orbifolds and classification

- Theorem: Any 2-orbifold is obtained from a smooth surface with corner by silvering some arcs and putting cone-points and corner-reflectors.
- A 2-orbifold is classified by the underlying smooth topology of the surface with corner and the number and orders of cone-points, corner-reflectors, and the boundary pattern of silvered arcs.
- proof: basically, strata-preserving isotopies.
- In general, a smooth orbifold has a smooth topological stratification and a triangulation so that each open cell is contained in a single strata.
- Smooth topological stratifications satisfying certain weak conditions have triangulations.
- One should show that the stratification of orbifolds by orbit types satisfies this condition.

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Existence of locally finite good covering

- Let X be an orbifold. Give it a Riemannian metric.
- There exists a good covering: each open set is connected and charts have cells as cover and the intersection of any finite collection again has such properties.
- Each point has an open neighborhood with an orthogonal action.
- Now choose sufficiently small ball centered at the origin so that it has a convexity property. (That is, any path can be homotoped into a geodesic.)
- Find a locally finite subcollection.
- Then intersection of any finite collection is still convex and hence has cells as cover.

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Covering spaces of orbifold

- Let X' be an orbifold with a smooth map $p : X' \to X$ so that for each point x of X, there is a connected model (U, G, ϕ) and the inverse image of $p(\psi(U))$ is a union of open sets with models isomorphic to (U, G', π) where $\pi : U \to U/G'$ is a quotient map and G' is a subgroup of G. Then $p : X' \to X$ is a *covering* and X' is a *covering orbifold* of X.
- Abstract definition: If X' is a (X_1, X_0)-space and $p_0 : X'_0 \to X_0$ is a covering map, then X' is a *covering orbifold*.
- We can see it as an orbifold bundle over X with discrete fibers. We can choose the fibers to be acted upon by a discrete group G, and hence a principal G-bundle. This gives us a regular (Galois) covering.

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Examples (Thurston)

- Y a manifold. $\tilde{Y}$ a regular covering map $\tilde{p}$ with the automorphism group Γ. Let $\Gamma_i, i \in I$ be a sequence of subgroups of Γ.
  - The projection $\tilde{p}_i : \tilde{Y} \times \Gamma_i \backslash \Gamma \to \tilde{Y}$ induces a covering $p_i : (\tilde{Y} \times \Gamma_i \backslash \Gamma)/\Gamma \to \tilde{Y}/\Gamma = Y$ where Γ acts by

$$\gamma(\tilde{x}, \Gamma_i \gamma_i) = (\gamma(\tilde{x}), \Gamma_i \gamma_i \gamma^{-1})$$

  - This is same as $\tilde{Y}/\Gamma_i \to Y$ since Γ acts transitively on both spaces.
  - Fiber-products $\tilde{Y} \times \prod_{i \in I} \Gamma_i \backslash \Gamma \to \tilde{Y}$. Define left-action of Γ by

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    We obtain the fiber-product

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$$\gamma(\tilde{x}, \Gamma_i \gamma_i) = (\gamma(\tilde{x}), \Gamma_i \gamma_i \gamma^{-1})$$

  - This is same as $\tilde{Y}/\Gamma_i \to Y$ since Γ acts transitively on both spaces.
  - Fiber-products $\tilde{Y} \times \prod_{i \in I} \Gamma_i \backslash \Gamma \to \tilde{Y}$. Define left-action of Γ by

$$\gamma(\tilde{x}, (\Gamma_i \gamma_i)_{i \in I}) = (\gamma(\tilde{x}), (\Gamma_i \gamma_i \gamma^{-1})), \gamma \in \Gamma.$$

    We obtain the fiber-product

$$(\tilde{Y} \times \prod_{i \in I} \Gamma_i \backslash \Gamma)/\Gamma \to \tilde{Y}/\Gamma = Y.$$

Developable orbifold

- We can let Γ be a discrete group acting on a manifold $\tilde{Y}$ properly discontinuously but maybe not freely.
- One can find a collection X_i of coverings so that
  - $\Gamma_i = \{\gamma \in \Gamma | \gamma(X_i) = X_i\}$ is finite and if $\gamma(X_i) \cap X_i \neq \emptyset$, then γ is in Γ_i.
  - The images of X_i cover $\tilde{Y}/\Gamma$.
- $Y = \tilde{Y}/\Gamma$ has an *orbifold quotient* of $\tilde{Y}$ and Y is said to be *developable*.
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Doubling an orbifold with mirror points

- A *mirror point* is a singular point with the stablizer group $\mathbb{Z}_2$ acting as a reflection group.
- One can double an orbifold M with mirror points so that mirror-points disappear. (The double covering orbifold is orientable.)
  - Let V_i be the neighborhoods of M with charts (U_i, G_i, ϕ_i).
  - Define new charts $(U_i \times \{-1, 1\}, G_i, \phi_i^*)$ where G_i acts by $(g(x, l) = (g(x), s(g)l)$ where $s(g)$ is 1 if g is orientation-preserving and -1 if not and ϕ_i^* is the quotient map.
  - For each embedding, $i : (W, H, \psi) \to (U_i, G_i, \phi_i)$ we define a lift $(W \times \{-1, 1\}, H, \psi^*) \to (U_i \times \{-1, 1\}, G_i, \phi_i^*$. This defines the gluing.
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Some Examples

- Clearly, manifolds are orbifolds. Manifold coverings provide examples.
- Let Y be a tear-drop orbifold with a cone-point of order n. Then this cannot be covered by any other type of an orbifold and hence is a universal cover of itself.
- A sphere Y with two cone-points of order p and q which are relatively prime.
- Choose a cyclic action of Y of order m fixing the cone-point. Then Y/Z_m is an orbifold with two cone-points of order pm and qm.

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Outline

Universal covering by fiber-product

- A universal cover of an orbifold Y is an orbifold $\tilde{Y}$ covering any covering orbifold of Y.
- We will now show that the universal covering orbifold exists by using fiber-product constructions. For this we need to discuss elementary neighborhoods. An *elementary* neighborhood is an open subset with a chart components of whose inverse image are open subsets with charts.
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Fiber-product for D^n/G_i

- If V is an orbifold D^n/G for a finite group G.
  - Any covering is D^n/G_1 for a subgroup G_1 of G.
  - Given two covering orbifolds D^n/G_1 and V/G_2, a covering morphism is induced by $g \in G$ so that $gG_1g^{-1} \subset G_2$.
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- Given coverings $p_i : V/G_i \to V/G$ for $G_i \subset G$ for V homeomorphic to a cell, we form a fiber-product.

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The construction of the fiber-product of a sequence of orbifolds

- Let $Y_i, i \in I$ be a collection of the orbifold-coverings of Y.
- We cover Y by elementary neighborhoods V_j for $j \in J$ forming a good cover.
- We take inverse images $p_i^{-1}(V_j)$ which is a disjoint union of V/G_k for some finite group G_k.
- Fix j and we form one fiber product by V/G_k by taking one from $p_i^{-1}(V_j)$ for each i.
- Fix j and we form a fiber-product of $p_i^{-1}(V_j)$, which will essentially be the disjoint union of the above fiber products indiced by the product of the component indices for each i.
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- Now, we wish to patch these up using imbeddings. Let $U \to V_j \cap V_k$. We can assume $U = V_j \cap V_k$ which has a convex cell as a cover.
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  - We form the fiber products of $p_i^{-1}(U)$ as before which can be realized in V_j and V_k.
  - Over the regular points in V_j and V_k, they are isomorphic. Then they are isomorphic.
  - Thus, each component of the fiber-product can be identified.
- By patching, we obtain a covering Y^f of Y with the covering map p^f.

Thurston's example of fiber product

- Let I be the unit interval. Make two endpoints into silvered points.
- Then $I_1 = I$ is double covered by $\mathbf{S}^1$ with the deck transformation group $\mathbb{Z}_2$. Let p_1 denote the covering map.
- $I_2 = I$ is also covered by I by a map $x \mapsto 2x$ for $x \in [0, 1/2]$ and $x \mapsto 2 - 2x$ for $x \in [1/2, 1]$. Let p_2 denote this covering map.
- Then the fiber product of p_1 and p_2 is what?
- Cover I by $A_1 = [0, \epsilon), A_2 = (\epsilon/2, 1 - \epsilon/2), A_3 = (\epsilon, 1]$.
  - Over A_1, I_1 has an open interval and I_2 has two half-open intervals. The fiber-product is a union of two copies of open intervals.
  - Over A_2, the fiber product is a union of four copies of open intervals.
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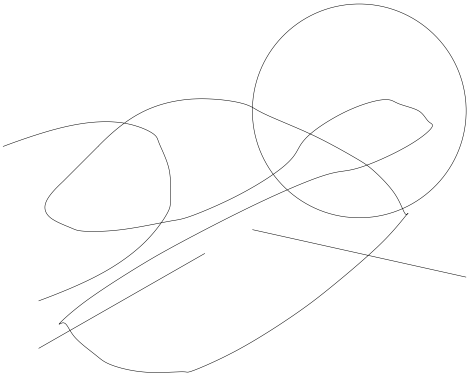
- By pasting considerations, we obtain a circle mapping 4-1 almost everywhere to *I*.



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The construction of the universal cover

- The collection of cover of an orbifold is countable upto isomorphisms preserving base points. (Cover by a countable good cover and for each elementary neighborhood, there is a countable choice.)
- Take a fiber product of Y_i, $i = 1, 2, 3, ...$ The fiber-product $\tilde{Y}$ with a base point $*$. We take a connected component.
- The for any cover Y_i, there is a morphism $\tilde{Y} \to Y_i$.
- The universal cover is unique up to covering orbifold-isomorphisms by the universality property.

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Properties of the universal cover

- The group of automorphisms of $\tilde{Y}$ is called the fundamental group and is denoted by $\pi_1(Y)$.
- $\pi_1(Y)$ acts transitively on $\tilde{Y}$ on fibers of $\tilde{p}^{-1}(x)$ for each x in Y. (To prove this, we choose one covering of Y from a class of base-point preserving isomorphism classes of coverings of Y. Then the universal cover with any base-point occurs will occur in the list and hence a map from $\tilde{Y}$ to it preserving base-points.)
- $\tilde{Y}/\pi_1(Y) = Y$.
- Any covering of Y is of form $\tilde{Y}/\Gamma$ for a subgroup Γ of $\pi_1(Y)$.
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- The group of automorphism is $N(\Gamma)/\Gamma$.
- A covering is regular if and only if Γ is normal.
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Induced homomorphism of the fundamental group

- Given two orbifolds Y_1 and Y_2 and an orbifold-diffeomorphism $g : Y_1 \to Y_2$. Then the lift to the universal covers $\tilde{Y}_1$ and $\tilde{Y}_2$ is also an orbifold-diffeomorphism. Furthermore, once the lift value is determined at a point, then the lift is unique.
- Also, homotopies $f_t : Y_1 \to Y_2$ of orbifold-maps lift to homotopies in the universal covering orbifolds $\tilde{f}_t : \tilde{Y}_1 \to \tilde{Y}_2$. Proof: we consider regular parts and model neighborhoods where the lift clearly exists uniquely.
- Given orbifold-diffeomorphism $f : Y \to Z$ which lift to a diffeomorphism $\tilde{f} : \tilde{Y} \to \tilde{Z}$, we obtain $f_* : \pi_1(Y) \to \pi_1(Z)$.
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Outline

Path-approach to the universal covering spaces.

- G-paths. Given an etale groupoid X. A G-path $c = (g_0, c_1, g_1, ..., c_k, g_k)$ over a subdivision $a = t_0 \leq t_1 \leq ... \leq t_k = b$ of interval $[a, b]$ consists of
  - continuous maps $c_i : [t_{i-1}, t_i] \to X_0$
  - elements $g_i \in X_1$ so that $s(g_i) = c_{i+1}(t_i)$ for $i = 0, 1, .., k - 1$ and $t(g_i) = c_i(t_i)$ for $i = 1, .., k$.
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- The two operations define an equivalence relation:
  - Subdivision. Add new division point t_i' in $[t_i, t_{i+1}]$ and $g_i' = 1_{c_i(t_i')}$ and replacing c_i with c_i', g_i', c_i'' where c_i', c_i'' are restrictions to $[t_i, t_i']$ and $[t_i', t_{i+1}]$.
  - Replacement: replace c with $c' = (g_0', c_1', g_1', .., c_k', g_k')$ as follows. For each i choose continuous map $h_i : [t_{i-1}, t_i] \to X_1$ so that $s(h_i(t)) = c_i(t)$ and define $c_i'(t) = t(h_i(t))$ and $g_i' = h_i(t_i) g_i h_{i+1}^{-1}(t_i)$ for $i = 1, .., k-1$ and $g_0' = g_0 h_1^{-1}(t_0)$ and $g_k' = h_k(t_k) g_k$.

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Compositions of G-paths

- All paths are defined on $[0, 1]$ from now on.
- Given two paths $c = (g_0, c_1, .., c_k, g_k)$ over $0 = t_0 \leq t_1 \leq ... \leq t_k = 1$ and $c' = (g'_0, c'_1, .., c'_{k'}, g'_{k'})$ such that the terminal point of c equals the initial point of c', the composition $c * c'$ is the G-path $c'' = (g''_0, c''_1, .., g''_{k+k'})$ so that
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  - $c''_i(t) = c_i(2t)$ for $i = 1, .., k$ and $c''_i(t) = c'_{i-k}(2t - 1)$ for $i = k + 1, ..., k + k'$.
  - $g''_i = g_i$ for $i = 1, .., k - 1$ and $g''_k = g_k g'_0$, $g''_i = g'_{i-k}$ for $i = k + 1, .., k + k'$.
- The inverse c^{-1} is $(g'_0, c'_1, ..., c'_k, g'_k)$ over the subdivision where $t'_i = 1 - t_i$ so that $g'_i = g^{-1}_{k-i}$ and $c'_i(t) = c_{k-i+1}(1 - t)$.

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- All paths are defined on $[0, 1]$ from now on.
- Given two paths $c = (g_0, c_1, .., c_k, g_k)$ over $0 = t_0 \leq t_1 \leq ... \leq t_k = 1$ and $c' = (g'_0, c'_1, .., c'_{k'}, g'_{k'})$ such that the terminal point of c equals the initial point of c', the composition $c * c'$ is the G-path $c'' = (g''_0, c''_1, .., g''_{k+k'})$ so that
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  - A homotopy class of c is denoted $[c]$.
  - $[c * c']$ is well-defined in the homotopy classes $[c]$ and $[c']$. Hence, we define $[c] * [c']$.
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  - The constant path $e_x = (1_x, x, 1_x)$. Then $[c * c^{-1}] = [e_x]$ if the initial point of c is x and $[c^{-1} * c] = [e_y]$ if the terminal point of c is y. Thus, $[c]^{-1} = [c^{-1}]$.

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- Seifert-Van Kampen theorem: X an orifold. $X_0 = U \cup V$ where U and V are open and $U \cap V = W$. Assume that the groupoid restrictions G_U, G_V, G_W to U, V, W are connected. And let $x_0 \in W$. Then $\pi_1(X, x_0)$ is the quotient group of the free product $\pi_1(G_U, x_0) * \pi_1(G_V, x_0)$ by the normal subgroup generated by $j_U(\gamma)j_W(\gamma^{-1})$ for $\gamma \in \pi_1(G_W, x_0)$ for j_U the induced homomorphism $\pi_1(G_W, x_0) \to \pi_1(G_U, x_0)$ and j_V the induced homomorphism $\pi_1(G_W, x_0) \to \pi_1(G_V, x_0)$.

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Examples

- Let a discrete group Γ act on a connected manifold X_0 properly discontinuously. Then (Γ, X_0) has an orbifold structure. Any loop can be made into a G-path $(1_x, c, \gamma)$ so that $\gamma(x) = c(1)$. and $c(0) = x$. Thus, there is an exact sequence

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  The fundamental group can be computed by removing open-ball neighborhoods of the cone-points and using Van-Kampen theorem.
- Suppose that a two-dimensional orbifold has boundary and silvered points. Then remove open-ball neighborhoods of the cone-points and corner-reflector points. Then the fundamental group of remaining part can be computed by Van-Kampen theorem by taking open neighborhoods of silvered boundary arcs. Finally, adding the open-ball neighborhoods, we compute the fundamental group.
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- We can obtain a 2-orbifold from a Seifert fibered 3-manifold M.
- X_0 will be the union of patches transversal to the fibers.
- X_1 will be the arrows obtained by the flow.
- The orbifold X will be a 2-dimensional one with cone-points whose orders are obtained as the numerators of the fiber-order.
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Covering spaces and the fundamental group

- One can build the theory of covering spaces using the fundamental group.
- Given a covering $X' \to X$:
  - For every G-path c in X, there is a lift G-path in X'. If we assign the initial point, the lift is unique.
  - If c' is homotopic to c, then the lift of c' is also homotopic to the lift of c provided the initial points are the same.
  - $\pi_1(X', x_0') \to \pi_1(X, x_0)$ is injective.
  - A map from a simply connected orbifold to an orbifold lifts to a cover. The lift is unique if the base-point lift is assigned. Thus, a simply connected cover of an orbifold covers any cover of the given orbifold.
  - From this, we can show that the fiber-product construction is simply-connected and hence is a universal cover.
  - Two simply-connected coverings of an orbifold are isomorphic and if base-points are given, we can find an isomorphism preserving the base-points.

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Covering spaces and the fundamental group

- One can build the theory of covering spaces using the fundamental group.
- Given a covering $X' \to X$:
  - For every G-path c in X, there is a lift G-path in X'. If we assign the initial point, the lift is unique.
  - If c' is homotopic to c, then the lift of c' is also homotopic to the lift of c provided the initial points are the same.
  - $\pi_1(X', x'_0) \to \pi_1(X, x_0)$ is injective.
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- A simply-connected covering of an orbifold X is a Galois-covering with the Galois-group isomorphic to $\pi_1(X, x_0)$.
- Proof: Consider $p^{-1}(x_0)$. Choose a base-point $\tilde{x}_0$ in it. Given a point of $p^{-1}(x_0)$, connected it with $\tilde{x}_0$ by a path. The paths map to the fundamental group. The Galois-group acts transitively on $p^{-1}(x)$. Hence the Galois-group is isomorphic to the fundamental group.

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The existence of the universal cover using path-approach

- The construction follows that of the ordinary covering space theory.
  - Let $\hat{X}$ be the set of homotopy classes $[c]$ of G-paths in X with a fixed starting point x_0.
  - We define a topology on $\hat{X}$ by open set $U_{[c]}$ that is the set of paths ending at a simply-connected open subset U of X with homotopy class $c * d$ for a path d in U.
  - Define a map $\hat{X} \to X$ sending $[c]$ to its endpoint other than x_0.
  - Define a map $\hat{X} \times X_1 \to \hat{X}$ given by $([c], g) \to [c * g]$. This defines a right G-action on $\hat{X}$. This makes $\hat{X}$ into a bundle.
  - Define a left action of $\pi_1(X, x_0)$ on $\hat{X}$ given by $[c] * [c'] = [c * c']$ for $[c'] \in \pi_1(X, x_0)$. This is transitive on fibers.
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