1 Introduction

About this lecture

- Notions of Inference
- Inference Rules
- Hypothetical Rules
- Derived Rules
- The Propositional Rules
- Equivalences
- We do up to Hypothetical Rules in Lecture 5.
- Course homepages: `http://mathsci.kaist.ac.kr/~schoi/logic.html` and the moodle page `http://moodle.kaist.ac.kr`
- Grading and so on in the moodle. Ask questions in moodle.

Some helpful references

- Richard Jeffrey, Formal logic: its scope and limits, Mc Graw Hill
- A mathematical introduction to logic, H. Enderton, Academic Press.
- Whitehead, Russel, Principia Mathematica (our library). (This could be a project idea.)
- `http://plato.stanford.edu/contents.html` has much resource. See "Realism, Informal logic 2. Deductivism and beyond," and "Nondeductive methods in mathematics."
- `http://ocw.mit.edu/OcwWeb/Linguistics-and-Philosophy/24-241Fall-2005/CourseHome/` See "Derivations in Sentential Calculus". (or SC Derivations.)
- `http://jvrosset.free.fr/Goedel-Proof-Truth.pdf` "Does Godels incompleteness prove that truth transcends proof?"

Some helpful references

- `http://en.wikipedia.org/wiki/Truth_table`,
- `http://logik.phl.univie.ac.at/~chris/gateway/formular-uk-zentral.html`, complete (i.e. has all the steps)
- `http://svn.oriontransfer.org/TruthTable/index.rhtml`, has xor, complete.

2 The notion of inference

The realism and antirealism

- If the tree fall in a forest, and no one was there to heard it, did it make a sound? (Berkeley)
- Realism believes in existence and independence of certain objects and so on. This is very close to logical atomism.
- Antirealism: One has to test to find out before it can considered to exists and so on.
- Since we do not know everything, which should we take as a position?

The notion of inference

- From a valid set of "assumptions" or "theorems" we wish to deduce more true statement.
- We give a collection of ten rules of inference that gives you true statements from assumptions. (The collection of rules depend on books but essentially equivalent.) (Some Postmordernist will call these just Rhetorics.)
- Inference = Deduction = Proof.
- This is actually weaker than TF table or truth tree method.
- If you take the antirealist's position, the deductions are only valid method. But we could also take the realist's position.
- The reason for doing it is that for Predicate calculus, TF methods cannot work since we have to check infinitely many cases. (incompleteness)

3 Nonhypothetical Inference Rules

Nonhypothetical Inference Rules

- Modus Ponens or condition eliminations ($\rightarrow E$). From a conditional and its antecendent, we can infer the consequent.
- $P, P \rightarrow Q, \vdash Q$.
- We can check in truth table.
- Example:
- $P, Q \rightarrow R, P \rightarrow Q, \vdash R$.
- We need two ($\rightarrow E$).

More rules

- Negation elimination ($\neg E$): $\neg\neg\phi \rightarrow \phi$.
- Conjunction introduction ($\wedge I$): $\phi, \psi \rightarrow \phi \wedge \psi$.
- Conjunction elimination ($\wedge E$) : $\phi \wedge \psi \rightarrow \phi, \psi$.
- Disjunction introduction ($\vee I$): $\phi \rightarrow \phi \vee \psi$ for any wff ψ.
- Disjuction elimination ($\vee E$): $\phi \vee \psi$, $\phi \rightarrow \chi$, $\psi \rightarrow \chi$. Then infer χ.
- Biconditional introduction.($\leftrightarrow I$): $\phi \rightarrow \psi$, $\psi \rightarrow \phi$. Then $\phi \leftrightarrow \psi$.
- Biconditional elimination. ($\leftrightarrow E$): $\phi \leftrightarrow \psi$. Then $\phi \rightarrow \psi$, $\psi \rightarrow \phi$.

Example 1

- $P \vdash (P \vee Q) \wedge (P \vee R)$.
- 1. P. Assumption
- 2. $P \vee Q$. 1. $\vee I$.
- 3. $P \vee R$. 1. $\vee I$.
- 4. $(P \vee Q) \wedge (P \vee R)$. 2.3. $\wedge I$.

Example 2

- $P, \neg\neg(P \rightarrow Q) \vdash (R \wedge S) \vee Q$.
- 1. P. Assumption
- 2. $\neg\neg(P \rightarrow Q)$. A.
- 3. $P \rightarrow Q$. 2. $\neg E$.
- 4. Q. 1.3. $\rightarrow E$.
- 5. $(R \wedge S) \vee Q$ 4. $\vee I$.

Example 3

- $P \vee P, P \rightarrow (Q \wedge R) \vdash R$.
- 1. $P \vee P$. A.
- 2. $P \rightarrow (Q \wedge R)$. A.
- 3. $Q \wedge R$. 1, 2. $\vee E$.
- 4. R. 3. $\wedge E$.

4 Hypothetical Rules

Hypothetical Rules

- Conditional introduction ($\rightarrow I$): Given a derivation of ϕ with help of ψ, we infer $\psi \rightarrow \phi$.
- Example:
- $P \rightarrow Q, Q \rightarrow R \vdash P \rightarrow R$. (Socrates is human, Humans are mortal, Thus, Socrates is mortal.)
- $P \rightarrow Q$. A.
- $Q \rightarrow R$. A.
- : P. H.
- : Q. 1,3, $\rightarrow E$.
- : R. 2.4, $\rightarrow E$.
- $P \rightarrow R$. 3-5. $\rightarrow I$.

Example

- $(P \wedge Q) \vee (P \wedge R) \vdash P \wedge (Q \vee R)$.
- 1. $(P \wedge Q) \vee (P \wedge R)$. A
- 2. : $P \wedge Q$. H
- 3. : P 2. $\wedge E$.
- 4. : Q 2. $\wedge E$.
- 5. : $Q \vee R$. 4. $\vee I$.
- 6 : $P \wedge (Q \vee R)$. 3.5. $\wedge I$.
- 7. $(P \wedge Q) \rightarrow (P \wedge (Q \vee R))$. 2-6 $\rightarrow I$.
- 8. : $P \wedge R$. H
- 9. : P 8. $\wedge E$.
- 10. : R 8. $\wedge E$.
- 11. : $Q \vee R$. 10. $\vee I$.
- 12 : $P \wedge (Q \vee R)$. 9.11. $\wedge I$.
- 13. $(P \wedge R) \rightarrow (P \wedge (Q \vee R))$. 2-6 $\rightarrow I$.
- 14. $P \wedge (Q \vee R)$. 1.7.13 $\vee E$.

Note

- Every hypothesis introduced begins at a new line.
- No occurance of a formula to the right of a vertical line may be cited after the line ended. (There may be multiple lines. See 4.20)
- If two or more hypothesis are ineffect, then the order that they are discharged is reverse.
- A proof is not valid until all the hypothesis is discharged.

Negation introduction

- Negation introduction ($\neg I$). Reductio ad absurdum, indirect proof.
- Given a derivation of absurdity from a hypothesis $\neg\phi$, we infer ϕ.
  - $\neg P \rightarrow P, \vdash P$.
  - 1. $\neg P \rightarrow P$. A.
  - 2. : $\neg P$. H (for $\neg I$)
  - 3. : P, 1.2, ($\rightarrow E$)
  - 4. : $P \wedge \neg P$. 2.3 ($\wedge I$).
  - 5. $\neg\neg P$. 2-4 $\neg I$.
  - 6. P. $\neg E$.

Example

- $P \rightarrow Q \vdash \neg P \vee Q$.
- 1. $P \rightarrow Q$.
- 2. : $\neg(\neg P \vee Q)$. H (for $\neg I$.)
- 3. ::P. H.(for $\neg I$).
- 4. :: Q. 1.3. ($\rightarrow E$)
- 5. :: $\neg P \vee Q$. 4 $\vee I$.
- 6. :: $(\neg P \vee Q) \wedge \neg(\neg P \vee Q)$. 2.5. $\wedge I$.
- 7. : $\neg P$. 3-6 $\neg I$.
- 8. : $\neg P \vee Q$. 7. $\vee I$.
- 9. : $(\neg P \vee Q) \wedge \neg(\neg P \vee Q)$. 2. 8 $\wedge I$.
- 10. $\neg\neg(\neg P \vee Q)$. 2-9 $\neg I$.
- 11. $\neg P \vee Q$. 10. $\neg E$.