Logic and the set theory

Lecture 3: Propositional Logic

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About this lecture

- Argument forms

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- Logical operators

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- Formalization: well formed formula (wff)

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  `http://mathsci.kaist.ac.kr/~schoi/logic.html` and the moodle page `http://moodle.kaist.ac.kr`

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- Grading and so on in the moodle. Ask questions in moodle.

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- `http://ocw.mit.edu/courses/linguistics-and-philosophy/24-241-logic-i-fall-2009/`

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- If P and Q, then R. It is not the case R. It is not the case P and Q.

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- $\vdash$ is used to mark the conclusion.

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- $(P \vee (Q \wedge (\neg R))) \rightarrow S$.

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- A subwff is a wff within a wff.
- As long as atomic sentence letters are well defined, there is no ambiguity in the meaning of wff.

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- $B \to \neg S$.

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- $\neg S \lor \neg B$.
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- If there were no God, then no movement is possible. But there are movements. Hence, God exists.

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- $\neg G \rightarrow \neg M, M, \vdash G$.

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- $(R \wedge S) \vee (S \wedge \neg R)$.

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- Each atomic formula has a truth or false value in a real world (or world A).
- Each wff has a truth or false value in a real world (or world A).
- This depends on the truth values of atomic formulas.

Truth tables

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  - One has to learn some notations... Sometimes use 0 and 1 instead of F and T.

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  - Every wff can be evaluated from this.
  - In computer science *xor*.

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- $(\neg G \to \neg M) \to (M \to G)$.
- Also compare $P \to Q$ and $\neg P \vee Q$. Check$(P \to Q) \leftrightarrow (\neg P \vee Q))$.
- This is used to compare.
- You can also use $\neg((P \to Q)$ xor $(\neg P \vee Q))$.

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- Given some formula, any assignment of T and F yields T in the truth table. Such a formula is said to be a *tautology*.

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- $P \wedge \neg P$.
- The formula which are not one of the above is said to be *truth-functionally contingent*.

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- $(\neg G \rightarrow \neg M) \rightarrow (M \rightarrow G)$.
- $(P \rightarrow Q) \leftrightarrow (\neg P \vee Q))$.
- $((P \rightarrow Q) \rightarrow R) \rightarrow (P \rightarrow R)$.

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- To check validity... We check when if every P_i is true, then so is Q.
- Or you can form $(P_1 \wedge P_2 \wedge \cdots \wedge P_n) \rightarrow Q$.

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- $R \to (P \leftrightarrow (P \vee (P \wedge Q)))$.