2.3. Applications of Linear Systems

GPS, Network Analysis, Electric Circuits, Balancing Chemical equations, polynomial interpolations

Global Positioning (GPS)

- Say earth radius is 1: $x^2+y^2+z^2=1$.
- A ship at (x,y,z) sends a signal to a satellite.
  - d = 0.469(t-t_0). Here d is the distance. 0.469 is the speed of light, t_0 the time sent by sat. and t the time received by ship.
  - $d=((x-x_0)^2+(y-y_0)^2+(z-z_0)^2)^{1/2}$. (x_0,y_0,z_0) is the position of the satellite.
  - Taking squares, we obtain $(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=0.22(t-t_0)^2$
  - We replace 0.22 to 1 for simplicity

Example

- Ship at (x,y,z), time t: unknown
- Satellites 1,2,3,4
- Data:

Satellite	Satellite Position	Time
1	(1,2,2)	1
2	(0,1,2)	2
3	(1,0,1)	1
4	(1,1,1)	2

Network Analysis

- A network: nodes (junction), branches
- We assume
  - One directional flow at a branch
  - Flow conservation at a node: the flow into the node equals the flow out.
  - Flow conservation of the network: The flow into the network equals the flow out.
  - See Example 2.
  - Example 3 (Liberty park traffic light)

Electric network

- Battery: pumps electrons : flow from + pole
- Volts: electric pressure, electrical potential
- Rate of flow: amperes
- Resistence: ohm: drops voltage

- Ohm's law: E=IR. E drop in voltage
- Kirchhoff's current law: flow in = flow out of a node
- Kirchhoff's voltage law: Any closed loop voltage drop = voltage rise

Examples

- Example 4: 9 volt, 4 ohm. Single circuit. Determine I. Use Voltage law. $4I=9$, $I=9/4$ A.
- Example 5: voltage 6V, 3V. Resistances: 1 ohm, 1 ohm, 1 ohm. Find currents I_1, I_2, I_3.
  - $I_1+I_2=I_3$ at A, $I_3 = I_1+I_2$ at B.
  - Left loop: $6 = I_1+I_3$
  - Right loop: $3+I_3 + I_2 = 0$
  - Outer loop: $3+6+I_2=I_1$

Balancing chemical equations

- $HCl + Na_3PO_4 \rightarrow H_3PO_3 + NaCl$
- The number of atoms must be preserved.
- $x_1(HCl) + x_2(Na_3PO_4) \rightarrow x_3(H_3PO_4) + x_4(NaCl)$
  - H: $1x_1 = 3x_3$
  - Cl: $1x_1 = 1x_4$
  - Na: $3x_2 = 1x_4$
  - P: $1x_2 = 1x_3$
  - O: $4x_2 = 4x_4$

Now solve this system…..

Polynomial Interpolation

- Given two points in a plane, find a 1^{st} degree polynomial whose graph passing through the two points:
- $y=ax+b$. (x_1,y_1), (x_2, y_2)
  - $y_1=ax_1+b$, $y_2 = ax_2+b$.
  - Consider a, b as variables
  - So $x_2y_1=x_2x_1a+ x_2b$, $x_1y_2 = x_1x_2 a + x_1b$. Subtract to get $x_2y_1-x_1y_2 = (x_2-x_1)b$. Thus $b = (x_2y_1-x_1y_2)/(x_2-x_1)$. To get a, just subtract.

n points in xy-plane, degree n-1 polynomial passing through…

- $(x_1,y_1), (x_2, y_2),\ldots,(x_n, y_n)$ distinct x coordinates
- $y=a_0+a_1x + a_2x^2+ \ldots+a_{n-1}x^{n-1}$.
- By substitutions:
  - $a_0+a_1x_1 + a_2x_1^2+ \ldots+a_{n-1}x_1^{n-1}=y_1$
  - $a_0+a_1x_2 + a_2x_2^2+ \ldots+a_{n-1}x_2^{n-1}=y_2$
  - ….
  - $a_0+a_1x_n + a_2x_n^2+ \ldots+a_{n-1}x_n^{n-1}=y_n$
- Since a_is are unknowns, our augmented matrix is:

Now use the augmented matrix

$$\begin{bmatrix} 1 & x_1 & x_1^2 & \ldots & x_1^{n-1} & y_1 \\ 1 & x_2 & x_2^2 & \ldots & x_2^{n-1} & y_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & x_n & x_n^2 & \ldots & x_n^{n-1} & y_n \end{bmatrix}$$

Example

- Find a cubic polynomial passing through:
- (-1,-1),(0,1),(1,3),(2,-1)

$$\begin{bmatrix} 1 & -1 & 1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 3 \\ 1 & 2 & 4 & 8 & -1 \end{bmatrix}$$

Ex. 2.3.

- 1-4 network problems
- 5-8 electric network
- 9-13 chemical balancing
- 14-16 interpolations
- T8: satellite
- T7: Integral approximation using interpolations